Small Representations of Integral Polytopes

A polytope is a bounded convex set $P \subseteq \mathbb{R}^n$ that can be represented as the intersection of finitely many halfspaces. Algebraically one can represent these halfspaces by linear inequalities. A polytope is called integral if all its vertices are integral, see Figure 1 for some examples. Integral polytopes play an important role in integer linear programming, because they can be used to model, e.g., colorings of graphs, machine schedules, or design decisions of networks, which has many real world applications.

In practice, however, such integral polytopes can be very complex, because exponentially many inequalities (halfspaces) are necessary to describe these objects or an inequality description may even be unknown. For this reason, one considers relaxations of integral polytopes $P \subseteq \mathbb{R}^n$, which are given by polytopes $Q \subseteq \mathbb{R}^n$ satisfying $P \cap \mathbb{Z}^n = Q \cap \mathbb{Z}^n$. That is, the integral points in Q are exactly those in P. Such a relaxation Q may be much smaller than P in terms of inequalities, see again Figure 1.

A recent research topic is to find the smallest number of inequalities in a relaxation of P. This smallest number is called the *relaxation complexity* of P. The goals of this bachelor project are

- to develop new models for computing the relaxation complexity;
- to implement the developed models and to compare them with existing methods from the literature;
- to evaluate the impact of additional constraints on the inequalities in a relaxation, e.g., how does the size of a relaxation grow if one does not allow arbitrary inequalities but inequalities with bounded coefficients.

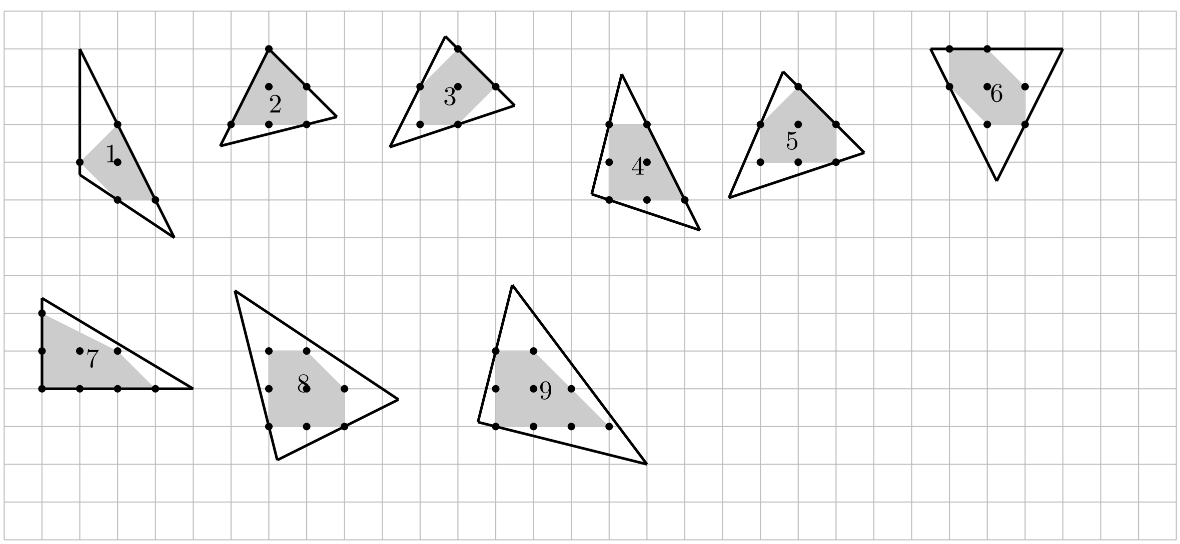

Figure 1: Nine integral polytopes (gray regions) and relaxations. All polytopes have a relaxation which is given by a triangle although some of the polytopes needs six inequalities in an exact description.