Finding Shortest Paths with Budget Constraints

Given a directed graph $G = (V, A)$ with arc weights $w\colon A \to \mathbb{R}_+$ as well as two distinguished nodes s, $t \in V$, the shortest path problem is to find a directed path consisting of arcs $a_1, \ldots, a_m$ connecting s and t that minimizes the path's weight $\sum_{i=1}^{m} w(a_i)$. This problem can be solved efficiently using dynamic programming techniques (Bellman-Ford Algorithm). In practice, however, a path connecting s and t may be required to satisfy further constraints, e.g., if we are interested in traveling from s to t as quickly as possible without spending more than a given amount of money. This leads to the constrained shortest path problem, which extends the classical one by introducing a cost function $c\colon A \to \mathbb{R}_+$ as well as a budget C. The aim is then to find a shortest path connecting s and t whose cost $\sum_{i=1}^{m} c(a_i)$ does not exceed the budget C. In contrast to the unconstrained shortest path problem, the constrained shortest path problem is known to be NP-hard.

In the literature, several methods have been discussed to solve the constrained shortest path problem. Lozano and Medaglia [1] propose a flow based algorithm, but also procedures based on dynamic programming exist. The goals of this bachelor project are

- to search the literature for dynamic programming based solution techniques,
- to compare these approaches with the algorithm suggested in [1], and
- to implement and test a dynamic programming algorithm as well as the algorithm from [1].

References

[1] L. Lozano and A. L. Medaglia, *On an exact method for the constrained shortest path problem*, Computers & Operations Research, 40 (2013), pp. 378–384.