Color Refinement: Shrinking Linear Programs

Given a matrix $A \in \mathbb{R}^{m \times n}$, a vector $b \in \mathbb{R}^m$, and a vector $c \in \mathbb{R}^n$, a linear program aims to minimize the linear function $c^\top x$ over all points $x \in \mathbb{R}^n$ satisfying $Ax = b$, $x \geq 0$, i.e., to solve $\min\{c^\top x \ : \ Ax = b, \ x \geq 0\}$. Although linear programs can be solved in polynomial time, practical instances may still be challenging to solve, for example, because of an enormous number of variables and/or constraints. To reduce the size of linear programs, Grohe et al. [1] developed a method that partitions the constraint system into smaller subsystems, so-called color refinement. This partition can be used to find a linear program having smaller dimension and that is equivalent to the original program. The hope is that the smaller program can be solved faster than the original one, and thus, to speed-up solving large scale linear programs.

The goals of this bachelor project are

- to understand the techniques described in [1] and
- to implement and test the color refinement algorithm on practical instances.

References

[1] M. Grohe, K. Kersting, M. Mladenov, and E. Selman, *Dimension reduction via colour refinement*. https://arxiv.org/abs/1307.5697, 2013.