Finding new connections of Godsil-McKay switching

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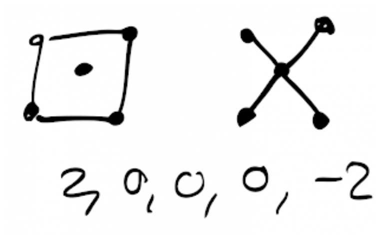
Summary

Spectral graph theory studies the relation between structural properties of the graph and the eigenvalues of associated matrices. Graphs are often studied by their adjacency matrix. Consider the following two graphs:



It is easy to check that the adjacency matrices of the above graphs have the same spectrum, however the graphs are clearly nonisomorphic. Two graphs with the same spectrum for some type of matrix are called *cospectral* with respect to the corresponding matrix. The spectrum contains a lot of information of the graph, but in general it does not determine the graph (up to isomorphism).

Cospectral graphs help us understand the weaknesses in identifying structures only using the spectrum. If a graph is not determined by the spectrum, this can be proved by constructing a nonisomorphic cospectral mate. Several tools for constructing cospectral graphs are known to exist, the most used is due to Godsil and McKay [1].

Similar methods (not yet linked) to Godsil-McKay switching seem to have appeared in the literature but in a very different context: to generate new Hadamard matrices [2]. A *Hadamard matrix* is a square matrix whose entries are either 1 or -1 and whose rows are mutually orthogonal. The work in [2] presents several operations that switch substructures of Hadamard matrices with the aim to produce new, generally inequivalent, Hadamard matrices. These operations have applications to the enumeration and classification of Hadamard matrices.

Details and general food for thought

- Perform a literature study on the switching methods from [1] and [2].
- Study the relationship between the methods stated in both papers.

References

[1] C. D. Godsil and B. D. McKay. Constructing cospectral graphs. *Aequationes Math.* 25 (1982), 257–268.

[2] W.P. Orrick. Switching Operations for Hadamard Matrices. *SIAM J. Discrete Math.* 22(1), 31-50.