On cospectral graphs

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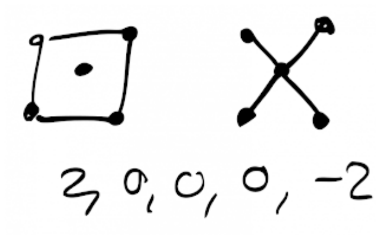
Summary

Spectral graph theory studies the relation between structural properties of the graph and the eigenvalues of associated matrices. Graphs are often studied by their adjacency matrix. Consider the following two graphs:



It is easy to check that the adjacency matrices of the above graphs have the same spectrum, however the graphs are clearly nonisomorphic. Two graphs with the same spectrum for some type of matrix are called *cospectral* with respect to the corresponding matrix. The spectrum contains a lot of information of the graph, but in general it does not determine the graph (up to isomorphism). So a central question is:

Given the spectrum of a graph, what can be said about its structure?

For example, we can see from the spectrum whether the graph is regular, or bipartite. Spectral graph theory looks at answering questions of this type.

Cospectral graphs help us understand the weaknesses in identifying structures only using the spectrum. If a graph is not determined by the spectrum, this can be proved by constructing a nonisomorphic cospectral mate. Several tools for constructing cospectral graphs are known to exist [3, 1, 5, 4]. However, there is no guarantee that such tools produce nonisomorphic cospectral graphs.

Details and general food for thought

- Perform a literature study on the existing tools to construct cospectral graphs.
- Study the graph switching proposed in [5].
- Find sufficient or necessary conditions to guarantee nonisomorphism for the cospectral graphs obtained using the switching method from [5].

References

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